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DO W_L AND H FORM A P-WAVE BOUND STATE?*ZHENG HUANG[†]

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ABSTRACT

We examine the possibility of bound state formation in the $W_L H \rightarrow W_L H$ channel. The dynamical calculation using the N/D method indicates that when the interactions among the Goldstone and Higgs bosons become sufficiently strong, a p -wave state [$I^G(J^P) = 1^-(1^+)$] may emerge.

We shall consider the elastic scattering of $W_L H$ and view the process as the dynamical force for the possible generation of bound states or resonances, of which W_L and H are constituents. To study the $W_L H$ scattering at high energies, it is much simpler to work with the Goldstone bosons (w^\pm, z) and H by invoking the Equivalence Theorem^{1,2,3} when away from the threshold. We assume that the electroweak symmetry-breaking sector can be effectively parameterized by the linear σ -model. It is sufficient to consider the $I_3 = 0$ channel $zH \rightarrow zH$ ($w^\pm H$ are similar). When gauge interactions are ignored, it is isolated and decoupled from other strong scattering channels. The Born amplitude for $zH \rightarrow zH$ is

$$\mathcal{T}^B(s, t, u) = -2i\lambda \left[1 + \frac{m_H^2}{s - M_Z^2} + \frac{3m_H^2}{t - m_H^2} + \frac{m_H^2}{u - M_Z^2} \right], \quad (1)$$

where $m_H^2 = 2\lambda v^2$ and $v = 246$ GeV; s , t and u are the Mandelstam variables: $t = -2\nu(1 - \cos \theta)$ and $u = (M_Z^2 + s_0 - s) + 2\nu(1 - \cos \theta)$, where $s_0 = 2m_H^2 + M_Z^2$ and ν is the CM momentum square.

Before we go on and discuss the dynamical feature of this scattering amplitude, some special attention has to be paid to the u -channel z -exchange. The matrix element is formally divergent at some scattering angle $\cos \theta = 1 + (s_0 - s)/(2\nu)$ when $(m_H + M_Z)^2 < s_0 \leq s$, at which $u - M_Z^2 = 0$. This is only possible when $m_H > 2M_Z$ and $s \geq s_0$. The first inequality is satisfied when $m_H > 2M_Z$ which implies that H is necessarily unstable. The nature of the singularity is logarithmic and can be seen in the p -wave amplitude

$$a_1^B = \frac{-\lambda}{16\pi} \left[\frac{2m_H^2}{2\nu} + \frac{3m_H^2(2\nu + m_H^2)}{4\nu^2} \ln \frac{m_H^2}{4\nu + m_H^2} - \frac{m_H^2}{4\nu^2} (2\nu + s_0 - s) \ln \frac{s_0 - s}{4\nu + s_0 - s} \right]. \quad (2)$$

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The amplitude goes to negative infinity at the location of the singularity $s = s_0$, which seemingly represents a repulsive force. However, such a singularity is superficial. The origin of such a singularity can be traced back to the inconsistent treatment of the unstable Higgs boson: H could first decay into two z 's, one of which can subsequently combine the other initial state z into H again when $s \geq s_0$. The u -channel z -exchange thus represents a real process (actually two successive real processes) and the total cross section is formally divergent. The solution to this superficial singularity lies precisely on the fact that the Higgs boson is unstable. The logarithmic singularity is smeared by the large uncertainty in the Higgs mass position due to the finite width of the Higgs boson, and effectively the singularity does not exist. Our minimal prescription is to allow the Higgs boson to develop a complex energy due to the Higgs width (Γ_H), but nevertheless to retain the quasi-two-body structure of the amplitude (a more strict treatment would be the consideration The “on-shell” condition for an unstable Higgs boson is then

$$(E_H - i\omega)^2 - \nu = m_H^2 - im_H\Gamma_H, \quad (3)$$

where $E_H(\omega)$ is the real (imaginary) part of the energy in the CM system, and Γ_H the Higgs decay width. The modified Mandelstam variables \hat{s} , \hat{t} and \hat{u} , first introduced by Peierls⁴, are

$$\hat{s} = s - im_H\Gamma_H \left(1 + \frac{E_Z}{E_H}\right); \quad \hat{t} = -2\nu(1 - \cos\theta); \quad (4)$$

$$\hat{u} = (M_Z^2 + s_0 - s) + 2\nu(1 - \cos\theta) - im_H\Gamma_H \left(1 - \frac{E_Z}{E_H}\right) \quad (5)$$

where s is redefined as $(E_Z + E_H)^2 - m_H^2\Gamma_H^2/4E_H^2$. Note that the threshold value of s is $s_{\text{th}} \equiv s(\nu = 0)$ which is smaller than $(m_H + M_Z)^2$ for the unstable H . With these modifications, the partial wave amplitude becomes completely regular and the principal part of the p -wave Born amplitude is positive over the whole physical region, thus representing an attractive force.

We now examine whether the interaction may provide a dynamical driving force strong enough to lead to the formation of bound states. Our goal is to sum up a class of ladder diagrams, according to the requirement of unitarity. This is done most consistently by an N/D method in dispersion theory⁵. The full partial wave amplitude $a_1(s)$ must satisfy the elastic unitarity condition: $\text{Im}a_1(s) = -\sqrt{4\nu/s} a_1^* a_1$ ($s > s_{\text{th}}$). In the N/D method, $a_1(s)$ is written as $N(s)/D(s)$ where $N(s)$ has only left-hand cuts and $D(s)$ has only right-hand cuts. An once-subtracted dispersion relation may be written for $D(s)$

$$D(s) = 1 - \frac{(s - \mu^2)}{\pi} \int_{s_{\text{th}}}^{\infty} ds' \sqrt{\frac{4\nu(s')}{s}} \frac{N(s')}{(s' - \mu^2)(s' - s)} \quad (6)$$

where μ is the subtraction point chosen to be at the reduced mass of the system. Unlike the “bootstrap” approach where the bound state itself should be included in the cross channels, $N(s)$ is approximated by the principal part of the Born amplitude (1) involving only elementary fields (z and H) for the calculation of a loose bound state. If for some value $s = s_B$ ($0 < s_B < s_{\text{th}}$), $D(s)$ vanishes, it implies that the scattering amplitude $a_1(s)$ has a pole at s_B which can be interpreted as the mass location of a bound state.

Fig. 1. Calculated mass M_A versus the input parameter m_H . For comparison, $m_H + M_Z$ is also presented by the dotted line.

Our numerical calculation shows that $D(s)$ for the p -wave develops a zero only when $m_H \geq 1$ TeV which coincides with a well-known unitarity bound first obtained by Lee, Quigg and Thacker ². We shall call this bound state A_1 as opposed to the QCD counterpart which is now called a_1 . As a result of strong self-coupling, some interesting particle spectrum besides the Higgs boson may emerge. In our model, the u -channel z -exchange turns out to be very important since we also checked the result without the u -channel contribution and found no bound states. As for the s -wave, the presence of the s -channel contribution provides extra repulsive force so that bound states do not form. In Fig. 1, we show the calculated mass of A_1 , M_A , versus the parameter m_H . The binding energy $B_A \equiv (m_H + M_Z) - M_A$ is only modest about order of M_Z for $m_H \sim 1.5$ TeV. The linearity of the curve represents the fact that the binding energy is proportional to the square root of the strength of the self-coupling λ . The primary decay modes for such a bound state would be $W_L + (2W_L)_{s\text{-wave}}$ through a Higgs boson exchange. The width can, in principle, be determined from the coupling g_{AZH} which can be calculated from the residue of $a_1(s)$ at $s = M_A^2$. Such an axial vector state contributes to a negative value of the S parameter in the precision measurement. More detailed implications can be found in Ref. ⁶.

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2. References

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